MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

EME4076 – MECHANICAL VIBRATIONS

(ME)

09 FEBRUARY 2023
2.30 p.m. - 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of 12 pages with 4 Questions, excluding the cover page.
2. Attempt **ALL** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the Answer Booklet provided.

Question 1 (25 Marks)

Part 1

Figure Q1_1(a) shows a vertical spring with a point mass m. Figure Q1_1(b) shows the free-body diagram of the mass at time t, together with its kinematics.

The spring has a non-uniform factor of $k - q(x + \Delta l_m)$, where
k is a constant term,
q is a factor that is dependent on the extension of the spring beyond original length,
Δl_m is the default change in length due to the weight of the mass.
x is the change in length of the spring from the equilibrium position, not counting Δl_m ,
$\ddot{x}$ is the acceleration of the change in length, and l_0 is the original length of the spring.

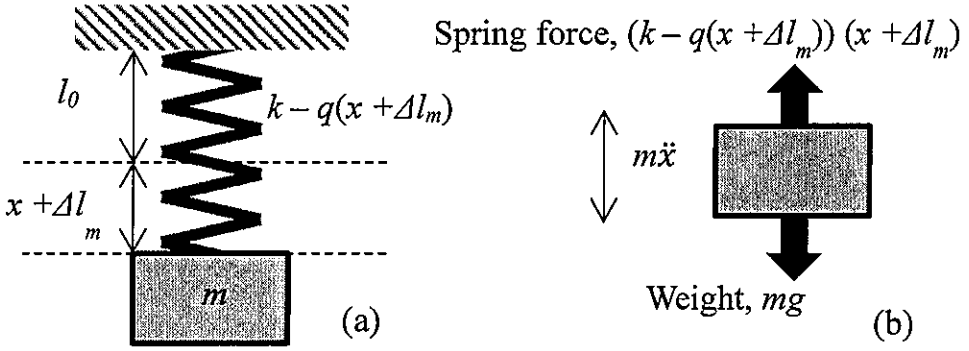

Figure Q1_1

a) Although the original length l_0 is not expected to be of significance in the kinematic analysis, it is still a physical limitation for the cyclic motions of this system. Briefly explain how this is so.

[2 marks]

b) Apply $\Sigma F = 0$ for the equilibrium position.
Find an equation for mg in terms of k, Δl_m and q.

[2 marks]

c) Let t be any moment in time.
Apply $\Sigma F = m\ddot{x}$ and present the answer as a homogeneous differential equation.
(Hint: there are no constant terms left.)

[4 marks]

d) Find the natural frequency of the system $\omega_{n,q}$ in terms of q, x, m, Δl_m and k.

[2 marks]

e) Consider the assumption of $x = Ae^{st}$, A and s being arbitrary constants.
Briefly explain how this assumption may not be applicable to this case.

[2 marks]

Continued...

Part 2

Figure Q1_2(a) shows a **top-down view** of a freely oscillating system with two springs of constants k_1 and k_2 , and a point mass m. Figure Q1_2(b) shows the free-body diagram of the mass at time t, together with its kinematics when it is moving into the first spring.
x is the displacement of the mass from the equilibrium position.
$\ddot{x}$ is the acceleration of the displacement.

This physical system is a 1-degree of freedom system, i.e. the springs are constrained along the direction shown. The point mass is sliding across a floor without friction.

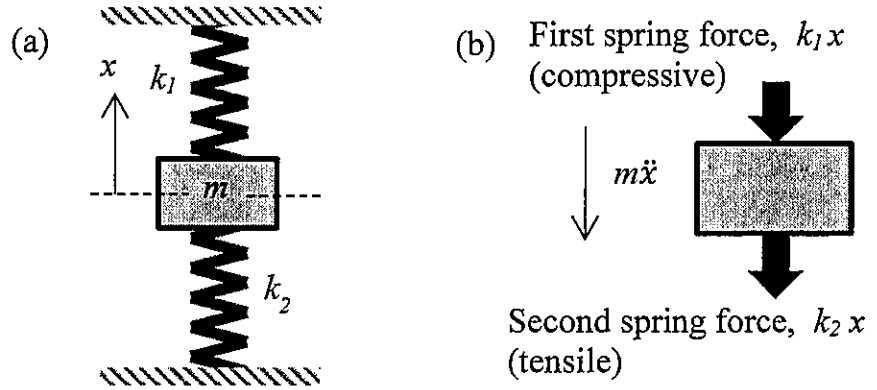

Figure Q1_2

a) The weight of the mass is not a significant factor in this system. Briefly explain this.
[1 mark]

b) Briefly explain why the forces and acceleration in Figure Q1_2(b) **must** be directed opposite of the direction of x in Figure Q1_2(a).
[2 marks]

c) Apply $\Sigma F = m\ddot{x}$ and present the answer as a homogeneous differential equation.
[2 marks]

d) Determine the natural frequency of the system ω_n in terms of k_1 , k_2 and m.
[1 mark]

e) Consider the response of this system (see Appendix for the equations).
Determine the equation for the times $t_{x,\max}$ when displacement is at its maximum magnitudes, in terms of $\dot{x}_0$, ω_n and x_o.
[3 marks]

f) Determine the equation $x_{\max}$ for the displacement at its maximum magnitudes in terms of $\dot{x}_0$, ω_n and x_o.
[2 marks]

g) From part f), determine the requirements, if any, for $\dot{x}_0$ and x_o so that the motion is possible.
[2 marks]

Continued...

Question 2 (25 Marks)

Part 1

Figure Q2_1 shows the schematic representation of a horizontal mass-spring system in one-degree translation. The system is underdamped. The mass ($m = 2$ kg) is in contact with the floor. The spring has a stiffness of $k = 2$ N/m.

The initial conditions are: $\dot{x}_0 = 0.2\ m/s$ and $x_0 = 0.2\ m$, in the same direction.

The damping coefficient, which is constant, is: $c = 2$ kg·rad/s.
Due to lubrication of the floor, the friction with the floor is insignificant and thus Coulomb damping can be disregarded.

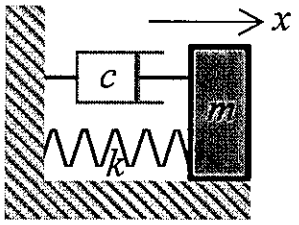

Figure Q2_1

a) The damper icon c in Figure Q2_1 represents the damping of the motion, but the damping is **not** due to a damping device within the system or sliding friction.
Briefly explain how this is so.
[1 mark]

b) Determine the natural frequency ω_n of this system.
[1 mark]

c) Determine the critical damping coefficient c_{cr}.
[1 mark]

d) Determine the damping ratio ζ.
Confirm that the system is underdamped.
[1 mark]

e) Determine the frequency ω_d of this system.
[1 mark]

f) Determine the time period T of this system.
[1 mark]

g) Determine the current phase angle ϕ_1 of this system.
[1 mark]

h) Determine the equation for velocity $\dot{x}$ in terms of t.
[2 marks]

i) Determine the first time t_{change} when the mass changes direction.
(*Hint: an initially negative answer means that a later time should be considered.*)
[3 marks]

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Part 2

Figure Q2_2 shows the displacement chart for a one-degree translational system that is underdamped.

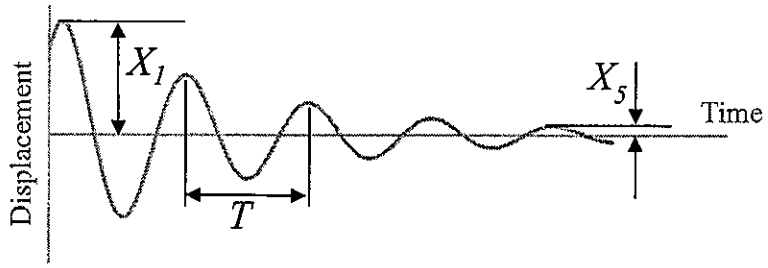

Figure Q2_2

The amplitude of the first peak is X_1 and the amplitude for the fifth peak is X_5. X_1 is 8 times the magnitude of X_5. These are the amplitudes that are readily measurable for this system.

The time period T has been measured as approximately 0.021 s. The mass of the system has been measured to be 0.861 kg.

a) Determine the logarithm decrement δ for this system.

[2 marks]

b) The logarithm decrement could have been determined from the first and second peak, but the second as well as third and fourth peaks are not measurable.
Briefly explain an example of the cause for this technical limitation.

[1 mark]

c) Determine the damping ratio of the system.
Also, confirm that the system is underdamped.

[3 marks]

d) Determine the natural frequency ω_n of the system.

[1 mark]

e) Although the time period T could be measured per cycle, the amplitude could not, as mentioned in part b).
Briefly explain an example of a reason for this disparity.

[1 mark]

f) Determine the equivalent stiffness k of the system.

[1 mark]

Continued...

g) The equivalent stiffness k of the system could have been determined before estimating for m, but this was not done.
Infer a practical reason for having measured m in order to determine k.

[2 marks]

h) Determine the critical damping coefficient c_{cr} .

[1 mark]

i) Determine the actual damping coefficient c .

[1 mark]

Continued...

Question 3 (25 Marks)

Part 1

There is an undamped one-degree translational system that is subjected to harmonic excitation; it is not in resonance. The equation of motion, in units of Newton, is as follows:

$$2\ddot{x} + 18x = 0.15\sin(2t)$$

The equation for system response is expected to be in the following form:

$$x(t) = x_f(t) + x_p(t)$$

Where x_f is the partial solution for a freely undamped vibrating system and x_p is the partial solution due to the harmonic excitation.

The initial conditions are $x_0 = 0.05$ m and $\dot{x}_0 = 0$.

a) From the equation of motion, determine the following (in their appropriate units):
   (i) Stiffness k
   (ii) Mass of the system m
   (iii) Frequency of the excitation ω
   (iv) Amplitude of the excitation F_O

[2 marks]

b) Determine the natural frequency ω_n .
   Confirm that the system is not in resonance.

[2 marks]

c) Determine the equation for $x_p(t)$.

[2 marks]

d) Determine the equation for velocity $\dot{x}(t)$; do not solve for unknown constants yet.

[1 mark]

e) Determine the full equations for $x(t)$ and $\dot{x}(t)$.
   (*Note: Apply the initial conditions.*)

[3 marks]

f) Confirm that neither ω_n or ω is the actual frequency of the vibration.
   (*Hint: Find the time periods from ω_n and ω first.*)

[3 marks]

Continued...

Part 2

There is an **undamped** one-degree translational system with an excitation; it is not in resonance. The equation of motion for this system is as follows:

$$2\ddot{x} + 8x = 5\sin t$$

The initial conditions are zero.

a) From the equation of motion, determine the following (in their appropriate units):
   (i) Stiffness k
   (ii) Mass of the system m
   (iii) Natural frequency ω_n

[2 marks]

b) Determine the function for the excitation force $F(t)$.

[1 mark]

c) Using convolution integral, determine the response equation for displacement $x(t)$.

[5 marks]

d) Determine the response equation for velocity $\dot{x}(t)$.

[1 mark]

e) Determine the maximum magnitude for displacement x_{max}.

[3 marks]

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Question 4 (25 Marks)

There is an undamped multi-degree system in horizontal translation, as shown in Figure Q4 (a). Figures Q4(b) and (c) show the free-body diagrams for m_1 and m_2. The effect of gravity is not significant in this case.

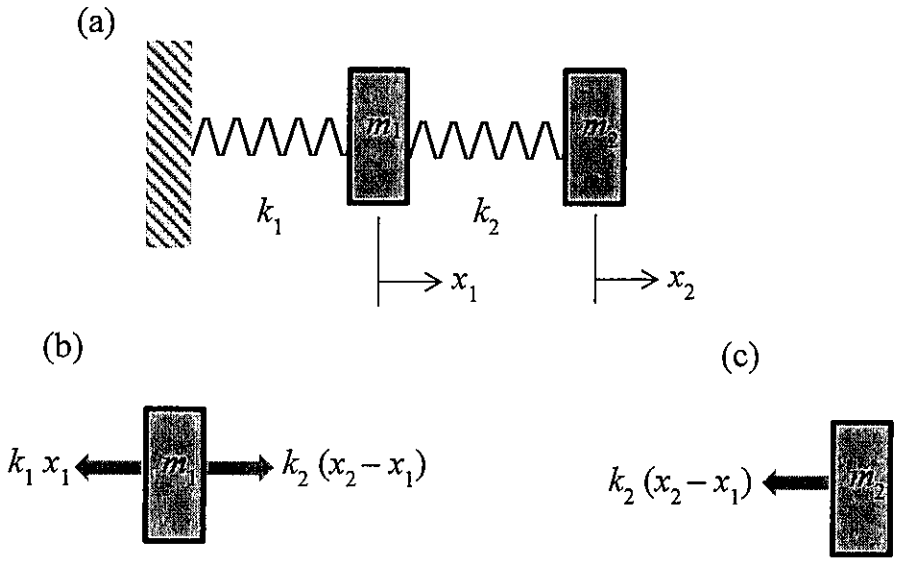

Figure Q4

The quantities in the system are as follows:

$m_1 = 4$ kg $\quad m_2 = 1$ kg $\quad k_1 = 16$ N/m $\quad k_2 = 20$ N/m

a) Using Figure Q4(a) and (b), determine the equations of motion for the system in linear form.
(*Note: include the values for the quantities that have been listed.*)

[2 marks]

b) Convert the equations of motion in part a) into matrix form.

[2 marks]

c) Determine the square root of the matrix for masses, $M^{-1/2}$.

[1 mark]

d) Determine the matrix product $KM^{-1/2}$.

[2 marks]

Continued...

e) Determine the mass-normalized stiffness matrix $\widetilde{K}$.

[1 mark]

f) Calculate the determinant for $\left(\widetilde{K} - \lambda I\right)$ in terms of λ, the eigenvalues for $\widetilde{K}$.

[2 marks]

g) Determine the eigenvalues λ_1 and λ_2 for $\widetilde{K}$.

[2 marks]

h) Determine the natural frequencies ω_{n1} and ω_{n2} of the system.

[2 marks]

i) Using eigenvalue λ_1, determine the eigenvector $v_{1,1}$ in terms of $v_{2,1}$.

[3 marks]

j) Using eigenvalue λ_2, determine the eigenvector $v_{1,2}$ in terms of $v_{2,2}$.

[3 marks]

k) Determine the actual magnitudes of $v_{1,1}$ and $v_{2,1}$.

[2 marks]

l) Determine the actual magnitudes of $v_{1,1}$ and $v_{2,1}$.

[2 marks]

m) Determine the matrix product $P^T M^{-1/2}$.

[1 mark]

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Appendix

Typical appearance of a homogeneous differential equation	$\ddot{x} + f\dot{x} + gx = 0$ Where f and g can be constants or functions of x, or both.
Natural frequency of a one-degree system in translation with constant stiffness	$\omega_n = \sqrt{\frac{k}{m}}$
Response of a free undamped oscillating one-degree system in translation and constant natural frequency	$x = \frac{\dot{x}_0}{\omega_n} cos(\omega_n t) + x_0 \, sin(\omega_n t)$ $\dot{x} = -\dot{x}_0 \, sin(\omega_n t) + x_0 \omega_n \, cos(\omega_n t)$ $\ddot{x} = -\dot{x}_0 \omega_n \, cos(\omega_n t) - x_0 {\omega_n}^2 \, sin(\omega_n t)$
Relevant inverse trigonometric identities	$\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$ $\cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}$
Critical damping coefficient	$c_{cr} = 2m\omega_n$
Damping ratio	$\zeta = \frac{c}{c_{cr}}$
Frequency of free underdamped one-degree system in translation and constant natural frequency	$\omega_d = \omega_n \sqrt{1-\zeta^2}$
Phase angle of a free underdamped one-degree system in translation and constant natural frequency	$\phi = \arctan\left(\frac{x_0 \omega_d}{\dot{x}_0 + x_0 \zeta \omega_n}\right)$
Response of a free underdamped one-degree system in translation and constant natural frequency	$x = e^{-\zeta\omega_n t}(C_1 \sin \omega_d t + C_2 \cos \omega_d t)$ $\dot{x} = e^{-\zeta\omega_n t}\big((C_2\omega_d + C_1\zeta\omega_n) \sin \omega_d t + (C_1\omega_d - C_2\zeta\omega_n) \cos \omega_d t\big)$ $C_1 = \frac{\dot{x}_0 + x_0\zeta\omega_n}{\omega_d} \qquad C_2 = x_0$

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Appendix (continued)

Damping force	$F_d = c\dot{x}$
Gravitational acceleration	$g = 9.81$ m/s^2
Logarithm decrement	$\delta = \frac{1}{n} ln \frac{X(t)}{X(t+nT)}$ $\delta = 2\pi \frac{\zeta}{\sqrt{1-\zeta^2}}$ $\delta = \zeta \omega_n T$
General equation of motion for an undamped one-degree system in translation with harmonic excitation	$m\ddot{x} + kx = F_0 \sin(\omega t)$
Particular equation for undamped one-degree system in translation with harmonic excitation	$x_p(t) = X \sin \omega t$ $X = \frac{F_0/m}{\omega_n^2 - \omega^2}$
Equation for an undamped one-degree system in translation, freely vibrating	$x_f(t) = A_1 \sin(\omega_n t) + A_2 \cos(\omega_n t)$
Convolution integral for an undamped system with arbitrary excitation	$x(t) = \frac{1}{m\omega_n} \int_0^t \big(F(\tau)\big) \sin\big(\omega_n(t-\tau)\big)\, d\tau$
Trigonometric identity for product of sines	$(\sin\theta)(\sin\varphi) = \frac{\cos(\theta-\varphi) - \cos(\theta+\varphi)}{2}$
Mass-normalized stiffness matrix	$\tilde{K} = M^{-1/2} K M^{-1/2}$
2-by-2 identity matrix	$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
Determinant of a matrix	$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\det A = ad - bc$
Matrix equation to obtain eigenvalues	$det\big(\tilde{K} - \lambda_n I\big) = 0$ n being the number of possible eigenvalues.

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Appendix (continued)

Quadratic solution

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Relation between eigenvectors and eigenvalues of a two-degree system

$$\left(\tilde{K} - \lambda_n I\right) \begin{bmatrix} v_{1,n} \\ v_{2,n} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$n = 1\,,\,2$

(Dimensionless) Magnitude of eigenvectors

$$\|v_n\| = \sqrt{\left(v_{1,n}\right)^2 + \left(v_{2,n}\right)^2} = 1$$

$n = 1\,,\,2$

Matrix of normalized eigenvectors for a two-degree system

$$P = \begin{bmatrix} v_{1,1} & v_{1,2} \\ v_{2,1} & v_{2,2} \end{bmatrix} \qquad P^T = \begin{bmatrix} v_{1,1} & v_{2,1} \\ v_{1,2} & v_{2,2} \end{bmatrix}$$

End of Paper